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## **Greek Astronomical Calendars and their Relation to the Athenian Civil Calendar**

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**Abstract:** Several investigations have been devoted to the Athenian calendar and to the cycles of Meton and Kallippos. However, most authors have not clearly distinguished between true and mean lunar months, nor between astronomical calendars and the Athenian calendar. In investigating the Athenian calendar, many authors have made use of the regular successions of full and hollow months described by Geminus in his *Isagoge*, without first making sure that these months were in actual use at Athens. Discussion as to whether 'the month' began with the astronomical New Moon or with the visibility of the crescent might have been avoided if the authors had realised that the word 'month' has several meanings and that in every particular case the meaning has to be inferred from the context. Peasants or soldiers, far away from civilisation, would start their month with the visible crescent, astronomers would make it begin at the day of true or mean New Moon, and cities would adapt their festival calendar to the needs of the moment, intercalating or omitting days in such a way that the festivals can be held at the days prescribed by law or tradition. Of course, it may happen any time that a civil month coincides with the astronomical or with the observed lunar month, but in absence of definite evidence we never have the right to identify a civil month with an astronomical month.

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## GREEK ASTRONOMICAL CALENDARS AND THEIR RELATION TO THE ATHENIAN CIVIL CALENDAR

SEVERAL investigations have been devoted to the Athenian calendar and to the cycles of Meton and Kallippos. However, most authors have not clearly distinguished between true and mean lunar months, nor between astronomical calendars and the Athenian calendar. In investigating the Athenian calendar, many authors have made use of the regular successions of full and hollow months described by Geminus in his *Isagoge*,<sup>1</sup> without first making sure that these months were in actual use at Athens. Discussion as to whether 'the month' began with the astronomical New Moon or with the visibility of the crescent might have been avoided if the authors had realised that the word 'month' has several meanings and that in every particular case the meaning has to be inferred from the context. Peasants or soldiers, far away from civilisation, would start their month with the visible crescent, astronomers would make it begin at the day of true or mean New Moon, and cities would adapt their festival calendar to the needs of the moment, intercalating or omitting days in such a way that the festivals can be held at the days prescribed by law or tradition. Of course, it may happen any time that a civil month coincides with the astronomical or with the observed lunar month, but in absence of definite evidence we never have the right to identify a civil month with an astronomical month.

In section 1, exact definitions of four different meanings of the word month will be given, which all occur in Greek literature. In sections 2–7, the astronomical calendars of Meton, Euktemon and Kallippos will be reconstructed, following the lines indicated by Scaliger and Fotheringham. It will be shown that these calendars were used by later astronomers like Aristarchos, Timocharis, and notably Hipparchos, for dating astronomical observations. The rules for counting days and the intercalation cycle will be reconstructed, as far as possible, from the astronomical texts.

In section 8, the relation of these calendars to the Athenian calendar will be discussed. It will be shown that the astronomical calendars differed from the Athenian festival calendar in many respects, and that there is no reason to assume that they ever were used at Athens.

### 1. FOUR KINDS OF MONTHS

Geminus gives, at the beginning of Chapter VIII of his *Isagoge*, the following definition: Month is the time from one conjunction to the next, or from one full moon to the next. He next defines the conjunction as the moment in which the sun and the moon are in the same degree of the zodiac.

Of course, Geminus knew very well that this astronomical definition is not fit for practical purposes, because the conjunction is not observable (except if there happens to be a solar eclipse). Therefore, he proceeds to describe how 'the ancients' named the days in accordance with the visible phases of the moon. 'The day, on which the moon appears anew, was called *νομήνια*', etc. He also says that solar eclipses take place on the last day of a month.

Strictly speaking, his two definitions are contradictory. According to the first definition, the month would begin at conjunction, and solar eclipses would take place on the first day of a month. According to the second definition, the month would begin with the first

<sup>1</sup> *Gemini Elementa astronomiae* (ed. Manitius; Lipsiae, 1898) 121.

visibility of the crescent, and solar eclipses would take place on one of the last days of a month. The former definition is astronomical, the latter practical.

In Babylonia, the month began on the evening on which the crescent was visible for the first time after New Moon. More precisely: If on the evening of the 29th day of any month the crescent is visible, the month has 29 days; if not, the month has 30 days. The same rule still holds in Muslim countries.

I shall call these months *observed lunar months*. The words of Geminus indicate that the Greek months originally were just observed lunar months.

The months beginning with the conjunction will be called *exact lunar months* or *conjunction months*. These months are a theoretical construction; they could not be used in practice in classical times, because before Kallippos (330 B.C.) astronomers were not able to predict the true conjunction. Still, Thukydides seems to use this kind of month in ii 28: 'During the same summer, on the first day of a month according to the moon (νουμηνία κατὰ σελήνην) the sun was eclipsed.' He adds that only on such a day a solar eclipse is possible.

The difference between the first days of an exact month and an observed lunar month is one or two days, or in exceptional cases three days.

If artificial months, alternating between 29 and 30 days, are counted off by fixed rules such as Geminus gives, these months will be called *mean lunar months*.

The difference between mean and exact lunar months is very small, for the largest difference between true and mean conjunction is, roughly speaking, half a day. In the time of Meton and Euktemon, astronomers were not yet able to calculate this difference. Even Eudoxos, sixty years later, explained the motion of the moon by means of three uniformly rotating spheres, which means that he assumed the motion of the moon in its orbit to be uniform. It was Kallippos who added two more spheres for the moon (and also for the sun). Still, Geminus, two centuries after Kallippos, makes no distinction between exact and mean lunar months; for, after having defined the exact lunar month he adds: 'The duration of a month is  $29\frac{1}{2} + \frac{1}{33}$  days.'

Therefore, in discussing the system of Meton and Euktemon and the passage from Thukydides, we shall not distinguish between mean and exact conjunction, nor between mean and exact lunar month.

Lastly, we have the months in actual use in the Greek cities. We shall call these months *civil months* or *months of the festival calendar*. Originally, these months must have been observed lunar months, but if a festival could not take place on the right day, the city officials had the right to intercalate days. This is proved by an Euboian law which allows an intercalation up to three days, if the stage technicians for a certain festival are late.<sup>2</sup>

Athenian inscriptions offer many examples of intercalated and second intercalated days. One inscription<sup>3</sup> even mentions a fourth intercalated Elaphebolion 9; so the Euboian maximum of three days does not hold for Athens.

The obvious conclusion from these and other facts is that the Athenian civil months did not coincide with the observed lunar months nor with the exact or mean lunar months of the astronomers.

The order of magnitude of the deviations may be judged from double dates 'according to the archon' and 'according to the god' in the second century B.C. Pritchett and Neugebauer found that the dates κατὰ θεόν are always higher than the dates κατ' ἄρχοντα, which shows that the archons often intercalated days. The differences range from 1 to 20 days (see n. 2).

The following table shows the order of magnitude of the differences between the starting-points of the four kinds of months:

<sup>2</sup> W. K. Pritchett and O. Neugebauer, *The Calendars of Athens* (Harvard Press, 1947) 20.

<sup>3</sup> W. K. Pritchett, 'Calendars of Athens Again', *Bull. de Corr. Hell.* lxxxi (1957) 269.

	Mean conjunction
$\frac{1}{2}$ day	True conjunction
1 or 2 days	Visible crescent
0 to 20 days	Beginning of civil month

## 2. FOTHERINGHAM'S RECONSTRUCTION OF THE CALENDARS OF EUKTEMON AND KALLIPPOS

In an important paper,<sup>4</sup> Fotheringham has reconstructed the methods of the schools of Meton and Euktemon, and of Kallippos, for counting years, months and days. Meton and Euktemon started their cycle with the observed summer solstice under the Athenian archon Apseudes:

432 B.C., June 27 = Phamenoth 21 = Apseudes Skirophorion 13.

This date is given by three independent witnesses and accepted by all chronologers. Kallippos started his cycle with the date:

330 B.C., June 28 = First year of Kallippos, Hekatombaion 1.

This date was obtained by Fotheringham by counting backward from the four dates of Timocharis according to the rules of Geminus. It was the day of the summer solstice according to Kallippos and according to modern calculation.

The rules for constructing the months of 30 and 29 days were described by Geminus as follows:

'The astronomers around Euktemon, Philippos and Kallippos . . . had found that 19 years contain 6,940 days, or 235 months. . . . They first assumed the 235 months to be of 30 days; this would give a total of 7,050 days. . . . The 7,050 days exceed the 6,940 days by 110. They now assume 110 months to be hollow, so that the 235 months of the 19-year cycle contain just 6,940 days. In order to obtain a uniform distribution of the omitted days they divided the number 6,940 by 110; this gives 63 days. Thus, every 63 days one day has to be designed as omitted in this cycle. It is not always the 30th day of the month which is omitted, but every time the one that falls after 63 days is omitted. In this cycle the months seem to have been correctly defined and the intercalary months distributed in accordance with the phenomena, but the length of the year has not been defined in accordance with the phenomena. For the length of the year, as a result of many observations, is agreed to be  $365\frac{1}{4}$  days, while the year resulting from the 19-year period is of  $365\frac{5}{8}$  days. These exceed the  $365\frac{1}{4}$  days by the 76th part of a day. Therefore, the astronomers around Kallippos corrected the difference and constructed the 76-year period consisting of four 19-year periods. It contains 940 months, of which 28 are intercalary, and of 27,759 days. They adopted the same arrangement of intercalary months. . . .'

Geminus says that every time after 63 days one day is to be omitted, but he does not say what is the first omitted day. Fotheringham assumed, quite naturally, that in both cycles the first 63 days were counted from the beginning of the cycle. Hence, in the cycle of Meton and Euktemon, the 110 days with numbers 64, 128, . . . up to 7,040 are omitted, and thus the 7,050 days were reduced to 6,940, as it ought to be. In the cycle of Kallippos, the 440 days with numbers 64, 128, . . . up to 28,120 would be omitted, and, moreover, the last day of the cycle has to be omitted in order to obtain the desired

$$28,200 - 441 = 27,759$$

<sup>4</sup> J. K. Fotheringham, 'The Metonic and Callippic Cycles', *Monthly Notices Roy. Astron. Soc.* 84 (1924) 383.

days. This arrangement would give a nearly uniform distribution of omitted days in both cycles.

We may call the omitted days 'omitted tithis', using the terminology of India.<sup>5</sup> Tithis are lunar days: 30 of them form a mean lunar month, and to 64 tithis correspond 63 civil days, so that roughly every 64th tithi has to be omitted.

Fotheringham tested his hypothesis by applying it to the dates of the four observations of Timocharis as given by Ptolemy:

- (1) Year 36 Poseideon 25 = Phaophi 16 = 295 B.C., December 20.
- (2) Year 36 Elaphebolion 15 = Tybi 5 = 294 B.C., March 9.
- (3) Year 47 Anthesterion 8 = Athyr 29 = 283 B.C., January 29.
- (4) Year 48 Pyanepsion 25 = Thoth 7 = 283 B.C., November 8.

Backward count from these four dates by the rules of Geminus led to one and the same epoch date in all four cases. This is a strong point in favour of Fotheringham's hypothesis that the first omitted day was the 64th day.

Fotheringham left no doubt that he considered the calendars of Meton and Kallippos as astronomical calendars only. He writes: 'It might be objected to my restoration of the Metonic cycle that the omission of every 64th day, irrespective of its place in the month, might lead to the omission of the proper days for important festivals, to which I reply that in all probability Meton never expected his calendar to be used for other than astronomical and meteorological purposes, though he probably knew that it would provide a standard by which the errors of the civil calendar could be measured.'

However, Pritchett and Neugebauer<sup>6</sup> doubted the existence of 'Greek astronomical months'. They supposed that Timocharis either dated his observations in the Greek festival calendar and that this calendar happened to be in accordance with the moon on these four days, or that 'astronomical records purposely disregarded arbitrary changes and kept their dates always *κατὰ θεόν*'.

Our first aim will be to show that Fotheringham was right in assuming that Timocharis used mean lunar months.

### 3. WHAT KIND OF MONTHS DID TIMOCHARIS USE?

Timocharis may have used (a) observed lunar months, or (b) conjunction months, or (c) actual Athenian months, or (d) mean lunar months. We shall now investigate these four possibilities.

(a) *Observed lunar months.* The third observation was on Kall. I 47 Anthesterion 8 = 283 B.C., January 29.

If the month were an observed lunar month, the crescent would have been visible on the evening of Anthesterion 0 = 283 B.C., January 21.

But the New Moon was on January 22 at 11 a.m. So the crescent could not have been visible, either in Athens or in Alexandria, on January 21.

(b) *Conjunction months.* The fourth observation was on Kall. I 48 Pyanepsion 25 = 283 B.C., November 8.

If the month were a conjunction month, the New Moon would have been on Pyanepsion 1 = 283 B.C., October 15.

However, the New Moon was on October 14 at 1 p.m. Hence, Timocharis did not use conjunction months.

<sup>5</sup> The fullest and clearest exposition of this subject has been given by Olaf Schmidt, 'On the Com-

putation of the Ahargana', *Centaurus* ii (1952) 140.  
<sup>6</sup> *Op. cit.* (n. 2) 20, 72 n. 9.

(c) *Athenian civil months.* In this case, we have to consider two possibilities.

(i) Many authors have assumed that the months in actual use at Athens were *observed* lunar months with occasional deviations due to careless observation or arbitrary intercalations. Now, careless observation of the crescent always causes *lower* date numbers, and so do intercalations of days. In fact, we have seen already that in the second century B.C. the dates *κατ' ἀρχοντα* were always lower than those *κατὰ θεόν*. However, the first three dates of Timocharis were all *higher* than the dates in an observed lunar calendar would be. Hence, if the Athenian calendar was an observed lunar calendar with or without arbitrary intercalations, the dates of Timocharis were not Athenian dates.

(ii) Dinsmoor and others have assumed that the Athenians actually used the *mean* lunar calendars of Meton and Kallippos, with occasional deviations due to carelessness or arbitrary intercalation of days. If this is assumed, and if Timocharis used this calendar, he actually used, in the four cases known to us, mean lunar months without any deviation. This leads us to the fourth possibility:

(d) *Mean lunar months.* No matter what point of view we may accept concerning the Athenian calendar, there is no escape from the conclusion that Timocharis used mean lunar months.

This was to be expected beforehand. If Timocharis wanted to use observed lunar months or the festival calendar, he would have had to ask a friend in Athens on what day the crescent had been seen, or how many days had been intercalated in this particular year. Moreover, as Fotheringham rightly remarks, civil dates would have been perfectly useless for later astronomers. On the other hand, if he used mean lunar months, calculated according to a simple rule, all Greek astronomers would have been able to convert his dates into any other calendar or to check his Egyptian dates.

What kind of mean months he used is clearly indicated by Ptolemy's own words: 'In the 36th year of the first Kallippian cycle.' The Kallippian cycle, as we know it from Geminus, was not only a definite method of counting years and months, but also of counting days.

Therefore, the most reasonable conclusion is that Timocharis used the system of Kallippos, as explained by Geminus and interpreted by Fotheringham.

#### 4. CONVERSION OF DATES

To make quite clear what I mean by 'the calendar of Euktemon' and the 'calendar of Kallippos', I shall now give easy rules for the conversion of dates from the Julian or Egyptian calendar into the calendars of Euktemon and Kallippos. Note that the Julian year  $-431$  is 432 B.C.

##### *A. Calendar of Euktemon*

(1) If a Julian or Egyptian date is given, we first determine the distance from the epoch of Meton and Euktemon  $-431$  June 27 = Phamenoth 21.

If we start with a Julian date, by far the safest and easiest way to find this distance is to find the Julian day numbers of the given dates by Schram's tables<sup>7</sup> and to subtract them. The difference is the *Euktemon day number* E, the number of elapsed days since the beginning of the cycle.

*Examples.* Consider the eclipse of  $-430$  August 3 mentioned by Thukydides, and the eclipse of  $-424$  October 9, which took place according to a Scholion to Aristophanes,

<sup>7</sup> R. Schram, *Kalendariographische und chronologische Tafeln* (Leipzig, 1908). These tables may be used for the Egyptian calendar too, but this is not so easy.



*Clouds* 584, under Stratokles in the month Boedromion. To find the Euktemon day numbers E.

	—430 August 3 (Thukydides)	—424 October 9 ( <i>Clouds</i> )
Julian day number	1564,215	1566,474
Epoch	—1563,813	—1563,813
Euktemon day number	E = 402	E = 2,661

(2) Divide E by 6,940. The quotient gives the number of elapsed cycles of 19 years. The remainder R gives the number of elapsed days of the current cycle.

(3) Divide R by 63. The quotient Q is the number of omitted tithis, and the sum  $R + Q$  is the number of elapsed tithis of the current 19-year cycle. Add 12. The sum  $T = R + Q + 12$  is the number of elapsed tithis, reckoned from Skirophorion 1.

(4) Divide T by 30. The quotient  $M = q$  is the number of the current month, Hekatombaion of the first year of the cycle being counted as month no. 1. The remainder  $r$  is the number of elapsed days in the month, so  $D = r + 1$  is the day number.

In our two examples, the calculation would be:

Eclipse of Thukydides	Eclipse of <i>Clouds</i>
E = 402	E = 2,661
R = 402	R = 2,661
Q = 6	Q = 42
+ 12	+ 12
T = 420	T = 2,715
M = q = 14	M = q = 90
D = r + 1 = 1	D = r + 1 = 16
(14th month, day 1)	(90th month, day 16)

### B. Calendar of Kallippos

(1) If a Julian date is given, we first determine the distance from the epoch —329 June 28, by subtracting the Julian day numbers obtained from Schram's tables. The difference is the *Kallippian day number* K, the number of elapsed days since the beginning of the cycle.

If an Egyptian date is given, we add to the number of the day the number of elapsed months of the current Egyptian year, multiplied by 30, and the number of elapsed Egyptian years since Thoth 0 of the year 330 B.C., multiplied by 365. Add 138, the distance between Thoth 0 and Hekatombaion 1 of the first year of Kallippos. The result of the addition is the Kallippian day number K.

For the dates of the four observations of Timocharis, the calculation in the Julian calendar would be as follows:

—294 December 20	—293 March 9	—282 January 29	—282 November 8
J = 1614,028	1614,107	1618,086	1618,369
—1601,069	—1601,069	—1601,069	—1601,069
K = 12,959	13,038	17,017	17,300

In the Egyptian calendar, the calculation would be:

36 II 16	36 V 5	47 III 29	48 I 7
16	5	29	7
30	120	60	0
12,775	12,775	16,790	17,155
138	138	138	138
<hr/> K = 12,959	<hr/> 13,038	<hr/> 17,017	<hr/> 17,300

(2) Divide K by 27,759. The quotient gives the number of elapsed cycles of 76 years. The remainder R gives the number of elapsed days of the current cycle.

(3) Divide R by 63. Add the quotient Q to R. The sum  $T = R + Q$  is the number of elapsed tithis or lunar days of the current cycle.

(4) Divide T by 30. The quotient  $q$  plus 1 gives the number of the month. The remainder  $r$  plus 1 gives the number of the day in the month:  $M = q + 1$ ,  $D = r + 1$ .

For the four observation dates of Timocharis, the calculation is as follows:

K = 12,959	13,038	17,017	17,300
R = 12,959	13,038	17,017	17,300
Q = 205	206	270	274
<hr/> T = 13,164	<hr/> 13,244	<hr/> 17,287	<hr/> 17,574
q = 438	441	576	585
r = 24	14	7	24
M = 439	442	577	586
D = 25	15	8	25

The month numbers may be compared with the month names given by Ptolemy in order to obtain information concerning the intercalation system of Kallippos. The day numbers agree exactly with those given by Ptolemy. I suppose that Timocharis made a calculation of the same kind in order to reduce his Egyptian dates to the calendar of Kallippos, which seems to have been in general use among astronomers of his time.

## 5. TWO METHODS OF COUNTING YEARS

For years after Kallippos Ptolemy always uses expressions like: 'At the end of the 50th year of the first Kallippian period' (*Almagest* iii 1, p. 207, Heiberg); 'In the 54th year of the second Kallippian period' (iv 11, p. 344); 'In the 17th year of the third Kallippian period' (iii 1, p. 195).

Ptolemy's direct source for these observations is Hipparchos. The months and days are given in the Egyptian calendar only. In addition to these dates, we have the four observations of Timocharis, with double dates in the Kallippian and Egyptian calendar.

For years before Kallippos, a different system is used. Ptolemy quotes from Hipparchos the details of three eclipse observations made at Babylon. The dates are given as follows (*Almagest* iv 11, pp. 340-3, Heiberg):

(1) Under the Athenian archon Phanostratos in the month Poseideon, in the night from Egyptian Thoth 26 to 27.

(2) Under the Athenian archon Phanostratos in the month Skirophorion in the night from Egyptian Phamenoth 24 to 25.



(3) Under the Athenian archon Euandros in Poseideon I in the night from Egyptian Thoth 16 to 17.

Several authors have supposed that the eclipses were observed in Athens and reduced to Babylon by Hipparchos or his source, but I have shown that this is astronomically impossible.<sup>8</sup> The observations were made in Babylon, and Hipparchos used the Athenian year and month names to make clear what months he meant. The original Babylonian reports gave, of course, years of Persian kings and Babylonian month names, but these would mean nothing to Hipparchos' Greek readers. Therefore, he was obliged to give the equivalents in at least one other calendar. Actually, he used the Athenian names of years and months, and the Egyptian date of the day.

Did Hipparchos use the Meton–Euktemon intercalation cycle, or the irregular Athenian intercalations of months? If he wanted to make it easy for himself and his readers, he had to use a regular cycle. According to Geminus, Kallippos considered the cycle of intercalary months introduced by 'those around Euktemon' as satisfactory, and adopted it himself. Therefore, we shall assume that Hipparchos used the same cycle. This hypothesis will be confirmed in two cases. In two more cases it does not lead to any inconsistency in the intercalation system, as Fotheringham has proved.

I suppose that Kallippos published a list of Athenian archons and of intercalary months up to his own time, and gave a rule how to continue the intercalation cycle after his own time. Hipparchos could use the list of archons for all years before Kallippos, and the Kallippian cycles for years after Kallippos. This would give him a consistent dating system for all dates from the time of Meton and Euktemon up to his own time.

Fotheringham's hypothesis may now be stated thus: Timocharis and Hipparchos both used this dating system, and Geminus also describes the same system. This hypothesis is confirmed by all the four Timocharis dates, and by the two Hipparchos dates for which we can check the intercalations, while the other Hipparchos dates do not contradict it. In my opinion, we may regard this hypothesis as firmly established.

## 6. INTERCALARY MONTHS

The distance between the last two observations of Timocharis is 283 days. Computed from the Athenian month names and day numbers, the distance would be 8 months and 17 days, i.e. 1 month less. The usual intercalary months in the Athenian calendar is a second Poseideon ( $\text{VI}_2$ ), so an Athenian intercalation cannot explain the difference of one month.

Scaliger, Mommsen and Fotheringham concluded from this that the year 6 in the cycle of Meton had a second Skirophorion ( $\text{XII}_2$ ). This is an additional argument in favour of the conclusion that Meton's cycle was an astronomical, and not an Athenian calendar. I shall not follow Ideler,<sup>9</sup> who corrected Pyanepsion in Ptolemy's text into Maimakterion. All manuscripts have Pyanepsion, and the manuscript tradition of Ptolemy's *Almagest* is very good.

The epoch of Kallippos was the first day of a Kallippian year, and since Kallippos adopted Meton's and Euktemon's intercalary months, the same epoch must also have been at the end of a Metonic year. The time between the two epochs was 5 cycles of 19 years plus 86 months, hence:

(1) The first 7 years of a Metonic cycle had 2 intercalations.

<sup>8</sup> B. L. van der Waerden, 'Drei umstrittene Mondfinsternisse bei Ptolemaios', *Museum Helveticum* xv (1958) 106. <sup>9</sup> Ideler, 'Zeitrechnung der Griechen und Römer', *Handbuch der klass. Altertumswiss.* 1892 i 741 A.

In the same way, the four observations of Timocharis can be used to count the number of months in a certain number of years of the cycle. The results are:

- (2) The first 4 years had 1 intercalation.
- (3) The first 5 years had 1 intercalation (hence the year 5 was normal).
- (4) The first 15 years had 5 intercalations.
- (5) The year 16 had a second Skirophorion.

From two solstice observations by Aristarchos at the end of the 50th year of the first Kallippian cycle, and by Hipparchos at the end of the 43rd year of the third Kallippian cycle (*Almagest* iii 1, p. 207, Heiberg), we derive:

- (6) The first 12 years had 4 intercalations.
- (7) The 19 years of the cycle had 7 intercalations.

The three Babylonian eclipses of Hipparchos confirm the results (6) and (7) and yield the following additional information:

- (8) The year 13 had a second Poseideon (VI<sub>2</sub>).
- (9) The first 11 years had 4 intercalations.

This information is consistent with the relation (6) obtained previously, for if the year 13 had a second Poseideon, the year 12 cannot have been intercalary, hence, if the first 12 years had 4 intercalations, the first 11 years already had 4 intercalations.

From (4), (6) and (8) we may conclude that the years 14 and 15 were normal.

## 7. DATES OF THE SUMMER SOLSTICE

Meton and Euktemon observed the summer solstice in the morning of Skirophorion 13; the Euktemon day number is 0. In order to obtain the summer solstice of the next year, Euktemon had to add  $365\frac{1}{4}$  days, and so on.

In order to obtain definite integer day numbers, we have to adopt a convention concerning the beginning of the day. I shall adopt the definition given by Geminus:<sup>10</sup> Day is the time from sunrise to sunrise. I shall also assume, as Ptolemy did, that the observed solstice was after sunrise. Now it is clear that we have to add 365 days for the first, second and third year, and 366 days for the fourth year. Thus, we obtain the Julian dates and the Euktemon days numbers E given in the second and third column of the table on the opposite page. The fourth column gives the quotient Q of the division by 63, the next one the number of elapsed tithis

$$T = E + Q + 12$$

and the last column the date in the calendar of Euktemon.

In the years 0, 4, 7, 9 and 11–16, where the intercalation is known, we see that the effect of the intercalation was to bring the solstitium date between XII 6 and XII 29. Assuming that the purpose of the intercalation was to avoid wide variations of this date, we may safely conclude that dates like XI 23 or XI 20 were not desired, and hence that the years 1 and 17 were normal. Just so, we may assume that dates like XII 15, XII 7, XII 18 and XII 11 for the years 3, 5, 6 and 8 were preferred to dates like I 15, I 7, I 18 and I 11 in the first month of the next year, and we may conclude that the intercalary years in the cycle of Meton were

2 or 3, 5, 8, 10 or 11, 13, 16, 18 or 19.

<sup>10</sup> *Gemini Elementa astronomiae* (ed. Manitius) Chapter 6, p. 68.

Year	Julian date	E	Q	T	Euktemon date	
0	-431 June 27	0	0	12	XII	13
1	-430 „ 27	365	5	382	XII (or XI)	23
2	-429 „ 27	730	11	753	XII or I	4
3	-428 „ 26	1095	17	1124	XII (or I)	15
4	-427 „ 27	1461	23	1496	XII	27
5	-426 „ 27	1826	28	1866	XII (or I)	7
6	-425 „ 27	2191	34	2237	XII (or I)	18
7	-424 „ 26	2556	40	2608	XII	29
8	-423 „ 27	2922	46	2980	XII (or I)	11
9	-422 „ 27	3287	52	3351	XII	22
10	-421 „ 27	3652	57	3721	XII or I	2
11	-420 „ 26	4017	63	4092	XII	13
12	-419 „ 27	4383	69	4464	XII	25
13	-418 „ 27	4748	75	4835	XII	6
14	-417 „ 27	5113	81	5206	XII	17
15	-416 „ 26	5478	86	5576	XII	27
16	-415 „ 27	5844	92	5948	XII <sub>2</sub>	9
17	-414 „ 27	6209	98	6319	XII (or XI)	20
18	-413 „ 27	6574	104	6690	XII or I	1

There remain three cases of doubt. If the three former years 2, 10 and 18 were intercalary, all solstitium dates would be in the last month of the year (XII or XII<sub>2</sub>, as the case may be). This was the hypothesis of Scaliger,<sup>11</sup> Em. Muller<sup>12</sup> and Fotheringham. If, on the other hand, one or two or three of the years 3, 11 and 19 were intercalary, one or two or three solstitium dates would be at the beginning of the next year (I 1, I 2 and I 4).

This is what Ideler, Böckh, Redlich and Gresswell have proposed.<sup>13</sup>

A list of solstitium dates like our last column would be indispensable for any Greek who wanted to use Euktemon's calendar in combination with Euktemon's parapegma. This parapegma<sup>14</sup> started with the summer solstitium and gave star phases and weather forecasts for specified numbers of days after the summer solstitium. A list of 19 dates of summer solstitia for the 19 years of a cycle would be necessary for anyone wishing to know the weather forecast for a particular day. Therefore, I am sure that Euktemon actually published such a list, annexed to his parapegma.

#### 8. THE CALENDAR OF EUKTEMON AND THE ATHENIAN CALENDAR

The differences between the calendar of Euktemon and the calendar of Athens are:

(1) Euktemon had a regular intercalation cycle of 19 years, whereas the intercalation in Athens was irregular.

(2) In Euktemon's calendar, after every 63 days one day was omitted, regardless of its position within the month. Thus it could happen that a month had no 15th day. In a festival calendar, this was, of course, impossible.

<sup>11</sup> Scaliger, *Em. Temp.* 72 (quoted from Ginzel ii 400).

<sup>12</sup> *Real-Enzykl.* i<sup>2</sup> (1862) col. 1049, s.v. 'Annus'.

<sup>13</sup> F. K. Ginzel, *Handbuch der Chronol.* ii 400.

<sup>14</sup> A. Rehm, *Griechische Kalender III, Das Parapegma des Euktemon* (Sitzungsber. Heidelberger Akad. (phil.-hist.) 1913, 3. Abh.).

(3) In the festival calendar, days could be intercalated, sometimes 1 or 2, sometimes even 4 days in succession. In Euktemon's calendar, there were no intercalated days.

(4) In Euktemon's calendar, the first day of a month would be the day of the mean *conjunction*, as Fotheringham has shown. On the other hand, the Greek months were originally meant to be observed lunar months, beginning with the *visible crescent*. This is clearly stated by Geminus (p. 105) and by Aratos (quoted by Geminus). The traditional names of the first and last days of the month clearly show that the first day was supposed to be the day of the crescent, and the last day the day of the conjunction.

Dinsmoor and other authors do not share this point of view. While admitting that the Athenians did not follow Euktemon's intercalation rules, Dinsmoor<sup>15</sup> still maintains that they began the month on the day of the (mean or true) conjunction, and that they used regular sequences of full and hollow months according to one of the rules explained by Geminus.

The only evidence for this hypothesis, as far as I can see, is the statement of Thukydides quoted in section 1. Thukydides says: 'On the first day of a month, according to the moon, the sun was eclipsed.' I accept Dinsmoor's translation of *νουμηνία* by 'on the first day of a month'. However, Thukydides adds the words *κατὰ σελήνην*, thus underlining that the months ought to be taken 'according to the moon', and not according to the archon.

In other passages, too, Thukydides avoids the use of the Athenian festival calendar for dating purposes. The reason was, as he himself explains (v 20), that this calendar was too irregular. He prefers expressions like 'at the end of the winter' or 'when the summer had just begun', and he claims that this system has the advantage, over dating by archons and other magistrates, that it enables him to give the length of the war exactly, within a limit of a few days.

Thukydides' attitude towards the beginning of the months seems to be much the same as towards the beginning of the seasons. He used neither the irregular months of the festival calendar nor the observed lunar months beginning with the visible crescent. He preferred to use lunar months beginning at the conjunction. I suppose he did not distinguish between the mean and the true conjunction. This would explain his assertion that solar eclipses always take place on the first day of a month according to the moon.

If this explanation is accepted, Thukydides' lunar months have nothing to do with the Athenian or any other city calendar. Therefore, Dinsmoor's only argument for his assertion that the Athenians used mean lunar months beginning with the (mean) new moon, loses its force.

On the other hand, the difficulties to which Dinsmoor's hypothesis gives rise are numerous:

(1) If the cycles of Meton-Euktemon and Kallippos were meant as cycles for the Athenian calendar, the intercalary month ought to be a second Poseideon in all cases. As we have seen in section 6, this cannot be brought into accord with the text of Ptolemy. This difficulty was 'solved', as we have seen, by correcting Pyanepsion into Maimakterion.

(2) If the Athenians followed the rules of Euktemon or Kallippos, there ought to be no intercalary days. To explain the existence of these days, it was supposed that the calendar sometimes got out of step with the moon and that days were intercalated to correct the error. Now how would this work?

A mean lunar calendar may get out of step with the moon if we forget to omit a day where the rule prescribes it. The obvious remedy would be to omit a day in a later month, not to intercalate days.

Another possibility would be to simplify the rules by assuming every second month to be hollow. This would give us too many hollow months, and the remedy would be to

<sup>15</sup> W. B. Dinsmoor, *The Archons of Athens* 314, 321.

intercalate days. If this is the explanation, we should expect 3 intercalary days in 8 years, i.e. 1 pro mille of the days would be intercalary.

However, Pritchett<sup>16</sup> found that 16 out of 275 decrees, ranging in time from 346 to 100 B.C., were passed on intercalated days. This is more than 5 per cent. Hence, only a minute part of the intercalations actually found can be explained as corrections of errors.

A third possibility of committing errors would be to begin a new month on the day of the visible crescent instead of beginning it at the astronomical new moon. The remedy would be to omit 1 or 2 days at the end of the next month and to start the new month on the day of conjunction. Hence, the correction of errors cannot explain the numerous intercalations of days.

(3) Pritchett and Neugebauer found differences between *κατὰ θεόν* and *κατ' ἄρχοντα* dates in the second century B.C. ranging from 1 to 20 days. Assuming that the dates 'according to the god' were mean or true lunar dates, it follows that the calendar 'according to the archon' was highly irregular and did not follow astronomical rules. Hence, Dinsmoor's hypothesis cannot be maintained for the second century B.C.

(4) Pritchett's investigation shows that intercalary days were just as frequent in the third as in the second century. Hence, Dinsmoor's hypothesis has to be abandoned also for the third century.

(5) Dinsmoor's hypothesis that the Athenian calendar was regular would lead to the conclusion that the duration of the prytanies was variable. However, Aristotle states that (in a normal year of 354 days) the first four prytanies have 36 days and the remaining six 35 days each.<sup>17</sup> So we have to give up Dinsmoor's theory for the fourth century as well, unless we pretend that Aristotle was misinformed on the prytany calendar of the city he was living in.

(6) Aristophanes complains that the Athenians 'confused the days up and down' and that the Gods 'go to bed without supper, not obtaining festival banquets duly on festival days'. This shows that the festival calendar was irregular in the late fifth century too.

(7) This conclusion is confirmed by five examples of differences between the calendars of different cities, ranging from 2 to 8 days.<sup>18</sup> There is no reason to suppose that the Athenian calendar was correct and the others in disorder.

*Conclusion.* Dinsmoor's hypothesis is not supported by any fact, and is contradicted by all the facts.

#### 9. THE SUMMER SOLSTICE ON THE MILETUS PARAPEGMA

One of the two parapegma fragments found in Miletus gives the date of a summer solstice under . . . *ΥΚΛΕΙΤΟC* as Skirophorion 14 = Payni 11. Now the true summer solstice was exactly at noon on 106 B.C. June 26 = Payni 11, and for every year earlier or later the shift in the Egyptian date is exactly  $\frac{1}{4}$  day. Hence, if the error of the observation does not exceed 2 days, the years 114 to 98 B.C. are possible.

I shall accept the opinion of Diels<sup>19</sup> that . . . *ΥΚΛΕΙΤΟC* must be Athenian archon Polykleitos. From historical sources we know that Polykleitos held office in some year at the end of the second century, so the astronomical and the historical evidence agree well.

Diels restricted the investigation to the years from 113 to 109 B.C. and found that the year of the observation was 109 B.C. If we accept this year and convert the Egyptian date Payni 11 into the calendar of Kallippos, we obtain

—108 June 26 = Kall. cycle III, year 69, Skiroph. 12.

<sup>16</sup> W. K. Pritchett, 'Calendars of Athens Again', *Bull. de Corr. Hell.* lxxxi (1957) 276.

<sup>17</sup> Arist. *Aθ. Πολ.* x 2.2.

<sup>18</sup> W. K. Pritchett, 'Julian Dates and Greek

Calendars', *Classical Philology* xlii (1947) 235. The differences are 8, 7, 2, 2 and 5 days.

<sup>19</sup> H. Diels and A. Rehm, *Paraepmenfragmente aus Milet* (Sitzungsber. Berliner Akad. 1904) 92.



This is not in accordance with the text date XII 14. The Euktemon date would be XII 8, still worse. The new moon was on June 14 about 8.40 a.m., so the exact lunar date would be XII 13, and the observed lunar date XII 11. The date number according to the archon would be 11 or 12 if the archon had not intercalated days, and less if he had. The differences between *κατὰ θεόν* and *κατ' ἄρχοντα* dates are positive or zero in all instances known from the second century B.C. So we find no explanation for the figure 14 engraved on the parapegma.

A solution of these difficulties may be found as follows. Among the years 114–98 B.C. there is just one year in which the Egyptian Payni 11 coincided with Skirophorion 14 according to the calendar of Kallippos, viz. the year 106 B.C. As we have seen, the summer solstice of this year fell just on Payni 11, or June 26. Hence the year 106 B.C. would fit extremely well in every respect.

If this year is correct, the writer of the Milesian parapegma would have used, just as all other astronomers did, the Kallippian calendar alongside the Egyptian calendar.

#### 10. THE LUNAR ECLIPSE OF OCTOBER 425 B.C.

As we have seen, the eclipse of 425 B.C., October 9, took place, according to Euktemon's calendar, in the middle of the month IV (Pyaneption). However, a scholion to *Clouds* 584 tells us that this eclipse took place under Stratokles in Boedromion. Hence, either the scholiast used Euktemon's calendar and made a mistake in his calculation, or he knew from a contemporary source that the civil month, in which the eclipse took place, was Boedromion. This is what Dinsmoor and others have assumed, and I shall accept this assumption as being the most probable one.

Now, if we assume that this civil Boedromion coincided with the third lunar month of the year, and hence, that Hekatombaion 1 was approximately July 27, we get into serious difficulties (see Dinsmoor, *Archons of Athens* 333). The first difficulty is that Aristotle refers to a comet as having been visible in Gamelion under the archon Eukles at about the time of the winter solstice (427 B.C., December 29). The preceding new moon was on December 15, so if the month Gamelion began on December 15 or a few days later, Aristotle's statement would be in perfect order. This would give us June 10 for the new moon at the beginning of the next year (426/5). Hence, the year 426/5 would have 14 months (June 10 to July 27), a manifest impossibility.

The second difficulty is that the four years 425/4, 424/3, 423/2, 422/1 would be normal years, which would mean an extremely irregular intercalation. Of course, this is not quite impossible, but all this looks highly improbable.

The easiest and most probable solution of all the difficulties seems to be, to drop the assumption that the civil Boedromion coincided with the third lunar month of the year 425/4. If we suppose that 14 or more days had been intercalated in the first three months of the year, the eclipse would take place at the end of Boedromion, and all difficulties disappear.

A difference of 14 or 15 days between the festival and the lunar calendar would be of the same order of magnitude as the differences between the festival and the prytany calendar in the third century, or between dates *κατ' ἄρχοντα* and *κατὰ θεόν* in the second century. So there is nothing improbable in the assumption of a difference of 14 or more days.

Of course, I do not pretend that this explanation is certain. The only safe conclusion is that we really do not know when the year 425/4 began, and whether the four following years were normal or not. Dinsmoor's 'fixed point', the eclipse of 425 B.C., October 9, is not at all a fixed point of the Athenian calendar.

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